Journal of Media Economics, 22:211–238, 2009

ISSN: 0899-7764 print/1532-7736 online
DOI: 10.1080/08997760903375902

Circulations, Revenues, and Profits in a Newspaper Market with Fixed Advertising Costs

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This article investigates a model in which 2 newspapers compete between them for readers with differentiated preferences and advertise new products at a cost per reader that decreases as the circulation increases. The model can account for the empirical regularity that the revenues from advertising and the profits of the newspapers increase more than proportionally with the circulation. A complementary finding is that a larger number of potential advertisers lowers the profits of both newspapers.

The relevance of advertising as an economic activity is very large. The total worldwide expenditure on advertising in 2006 was $407 billion (PricewaterhouseCoopers, 2007), representing approximately 2% of total gross domestic product. Advertising also represents the largest source of revenues for many newspapers; for example, a study of the Inland Press Association & International Newspaper Financial Executives (2008), based on 2007 U.S. data, indicates that the average advertising revenue ranges between 60% and 70% of total revenue, depending on the newspapers' circulation.

In general, the revenues generated by advertising and the profits of the newspapers increase more than proportionally as the circulation increases (Brody & Picard, 1997; Inland Press Association, 2008). The purpose of this article is to show that such regularity can be accounted for by a model in which two newspapers compete between them for readers, in a standard Hotelling setting, with product differentiation, and publish the ads of the sellers of new products. The advertising cost per reader decreases as the circulation increases, due to a fixed cost which must be borne to advertise each single product. The presence of fixed costs in advertising is

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widely recognized in the literature; for example, Armstrong (2006, p. 681) acknowledged that "it is plausible that a platform incurs costs in dealing with each advertiser" (in addition to a cost proportional to the circulation). Besides transaction costs, our fixed cost could include the cost of enriching the newspaper's blueprint, to incorporate each new ad. An additional cost element, positively related to the newspaper's circulation, would not substantially modify the results.

Our model builds on the growing literature on "two-sided platforms" that provide content to an audience, and sell access to the same audience to the advertisers, surveyed in Anderson and Gabszewicz (2006), Armstrong, (2006), and Tirole and Rochet (2006). In particular, we focus on a setting in which each reader purchases only one newspaper while each producer can, in principle, place his or her ads in both newspapers. In the language of two-sided platforms, the readers are always "single-homing" while the advertisers are allowed to "multi-home." One of the two newspapers has an exogenous competitive advantage over the other, in the sense that its intrinsic features are a closer match for the preferences of a larger number of readers. In equilibrium, this advantage translates into a larger circulation, which reduces the incidence of the fixed advertising cost, in per-reader terms. The newspaper can then profitably publish the ads of a larger number of sellers, and realize advertising revenues more than proportionally larger than the revenues of the other newspaper.

The number of potential advertisers has an impact on the way in which the exogenous advantage is exploited by the newspaper enjoying it. If there are no advertisers, the newspaper's pricing strategy is "softened" to the largest possible extent by the heavier revenue losses driven by lower prices, given the larger potential circulation; hence, a relevant part of the exogenous circulation advantage is dissipated, in equilibrium. As the number of potential advertisers increases, price cuts aiming at increasing the circulation become more and more appealing, due to the possibility that they offer to lower the incidence of the fixed advertising cost, in per-reader terms, and to thereby increase the number of products that can be profitably advertised. This effect is stronger for the newspaper with an exogenous advantage in the circulation market, which can then ultimately retain a larger part of the same advantage. If the number of potential advertisers is relatively large, the same newspaper may even charge a lower price, and thereby increase its circulation, over and above the level reflecting its exogenous advantage.

The increased competition for readers lies behind a further result of the model: A larger number of potential advertisers translates into *lower* profits, for both newspapers. Essentially, a larger number of advertisers makes differentiation of the content a less effective shield for the newspapers' profits; this effect can more than offset the effect of the larger number of ads published. As we show later, this effect does not make the predictions of the model inconsistent with the trends currently observed in many newspaper markets, where advertisers are shifting from newspapers to other media, while at the same time the newspapers' profits are falling. In fact, the model can account for a simultaneous decrease both in advertising, and in the profitability of the newspapers, as far as the two effects are accompanied by a falling total circulation.

The tendency for newspapers with a larger circulation to charge lower prices for their ads, in per-reader terms, is already documented in the seminal article by Cover, Thompson, and Coheneur (1931). Picard (1998) showed that the relation between the circulation and the absolute prices of the ads may be highly nonlinear, primarily because of the impact of premium rates for newspapers with a very high circulation. The trade-off between the prices charged to the readers and the circulation, and their effects on the prices charged to the advertisers and the advertising revenue, are investigated by Corden (1953) and by Blair and Romano (1993), among others. The possibility of "circulation spirals" driven by positive feedbacks between circulation and advertising has also been investigated, in a dynamic environment, by Furhoff (1973) and by Gustafsson (1978). Different features of the newspapers, which can contribute to shaping the interactions between circulation and advertising, are reviewed in Reddaway (1963). The trade-off between the circulation and the proportion of high income-readers, within a model of vertical product differentiation, is empirically investigated by Thompson (1989).

Among the papers investigating specific factors that can lead to asymmetric outcomes, Ferrando, Gabszewicz, Laussel, and Sonnac (2008) investigated a setting where the newspapers' managers choose the prices charged to the readers and the advertisers taking into account the (rational) expectations previously formed by the members of the two groups. Unlike in our article, the newspapers are symmetrically located in the space of the readers' preferences, and the readers have heterogeneous attitudes toward ads. The model always features a symmetric equilibrium; despite the ex-ante symmetry, the model may also feature asymmetric equilibria at which one newspaper has a larger circulation and publishes a larger number of ads. Gabszewicz, Garella, and Sonnac (2007) investigated a dynamic model with two newspapers in which a larger number of ads published can make the respective newspaper more appealing, either because the readers are "ad lovers" or because the newspaper can use the higher advertising revenues to improve their quality. As in our article, one of the newspapers has an exogenous, potential circulation advantage, which effectively leads to a circulation spiral. The article focuses on the conditions under which such spiral actually leads to the elimination of the disadvantaged newspaper. Resende (2008) showed that the exogenous advantage of a newspaper representing the "voice of the majority" may be endogenously enhanced by advertising, if the readers are ad lovers.

The "profit squeeze" resulting from the increased competition for readers, as the number of the potential advertisers increases, is reminiscent of the findings in Gabszewicz, Laussel, and Sonnac (2001), where the newspapers are allowed to choose their locations in the product space, and a larger volume of advertising may induce a switch from an equilibrium with maximal differentiation and relatively high prices, to an equilibrium with minimum differentiation and a relatively low price.

The model is described in the next section. A subsequent section investigates a benchmark case with no advertising revenues, and provides a general characterization of the equilibria. Then, we investigate the equilibrium of the case with advertising, its comparative statics, and its welfare properties. The final section concludes the article with some suggestions for future research. All proofs are in the Appendix.

THE MODEL

Readers

There are a continuum of readers–buyers, or just "readers," with total mass $r > 0$, uniformly distributed over the interval [0, 1]. Newspapers, formally introduced later, also have an "address" in [0, 1]. Each reader's address identifies the reader's "ideal" newspaper as defined, for example, by the topics and the geographical areas covered, and by the editorialists' orientation. A reader $s \in [0, 1]$ derives utility $u - t|s - L| - p$ from reading a newspaper located at $L \in [0, 1]$, purchased at the price $p \in \mathbb{R}$. $u > 0$ and $t > 0$, respectively, measure the highest utility that a reader can possibly achieve, if he or she purchases a newspaper whose address coincides with his or hers, at a zero price, and the pace at which utility decreases, as the distance between the reader's and the newspaper's address increases. u is sufficiently large that each reader always purchases the newspaper that yields the higher utility. A reader who is indifferent between the two newspapers can purchase either one of them with probability $\frac{1}{2}$.

The number of ads published does not affect the readers' decisions concerning which newspaper to buy, nor the value of the ads for the sellers. The fixed cost of advertising, formally introduced later could, in principle, include the cost of the content that must be added to induce the readers to pay attention to the ads, rather than ignoring them—as they could do with relative ease, unlike, for example, TV viewers. A study of the German market for magazines by Kaiser (2006) found values of the parameters that were compatible with a "neutral" attitude toward advertising for readers representing 52.5% of the magazines' total circulation, whereas readers representing 40.7% of the magazines' circulation seem to appreciate advertising. If the readers valued ads positively, the effect associated with increases in advertising, investigated in this article and the effect studied in Resende (2008), would be mutually reinforcing.

Sellers

Our formalization of the demand for advertising builds on Anderson and Coate (2005) and Armstrong (2006). Ads can be placed by the monopolist producers–sellers of new products—or just "sellers." Each seller features a parameter $w \geq 0$, measuring the buyers' (common) willingness to pay for his or her product. For tractability, we assume that w is uniformly distributed over $[0, v]$, for an exogenous $v > 0$. The mass of the population of the sellers, $\mu \geq 0$, is exogenously given. Each buyer only purchases the products advertised in the newspaper of his or her choice; the possibility that only a fraction of the readers will actually purchase each product advertised can be formalized by suitably adjusting the value of μ. All sellers face a common, constant unit cost of production, which is set equal to zero without any further loss of generality. Each seller decides whether to advertise in each newspaper and sets the price of the respective product.

With a positive advertising cost, a lower bound of the support of the distribution of w equal to zero guarantees that the sellers whose products are associated with the lower values of w do not advertise in either newspaper, in equilibrium. If, by contrast, both newspapers accepted ads from all sellers, the newspapers' advertising revenues would be proportional to the circulation. However, situations in which the number of ads published would be unresponsive to changes in the advertising fee do not appear to be especially realistic.

Newspapers

There are two newspapers: A and B. A standard assumption in the literature is that the competing two-sided platforms are located at the opposite extremes of the space of the users' preferences (see, among others, Anderson & Coate, 2005; Armstrong, 2006; Laffont, Rey, & Tirole, 1998a, 1998b). In this article, by contrast, we introduce a tractable type of asymmetry by assuming that one of the two newspapers enjoys a "better location": B is located at 1, whereas A is located at a given $a \in \left(0, 15 - 6\sqrt{6}\right)$, where $15 - 6\sqrt{6} \approx 0.3031$ (see Figure 1). We thus assume, without loss of generality, that A is the "closer" newspaper for a larger fraction of the readers and is, therefore, better suited to match the preferences of such readers than B, in principle. The restriction to conveniently small values of a guarantees existence of an equilibrium in pure strategies in the case with no advertising, which provides a point of reference for the analysis of the cases with advertising.

The price per copy of newspaper $n \in \{A, B\}$, p_n, is set by the newspaper's manager—or, more synthetically, "by the newspaper." Prices below the cost per copy printed, and possibly even below zero, may be charged and are indeed charged, in equilibrium, as we see later. In general, newspapers do not offer cash to their readers, possibly due to the difficulty of verifying "how carefully" each reader reads the newspaper and, in particular, the ads published therein. However, the readers often receive goods or coupons or are invited to join clubs offering free gifts and special discounts, which can be worth more to them than the price of purchase of the newspapers. Some comments on the potential relevance of non-negativity constraints on prices are made later.

The advertising fee results from bargaining between each newspaper's manager and each single advertiser. We apply the generalized Nash bargaining solution with weights $\theta \in (0, 1)$ and $1 - \theta$; therefore, θ and $1 - \theta$ measure the fractions of the net gains from trade received by the newspaper and by the advertiser. This assumption, which requires the newspapers' managers to know the readers' willingness to pay for the advertisers' products, eliminates the inefficiencies potentially driven by the monopoly power that each newspapers enjoys, due to its position as an exclusive gateway to its readers. The assumption thereby eliminates relatively large, unexploited profit opportunities that could cast doubt on the relevance of the setting considered. The results presented in the article are, however, qualitatively similar to those that would be obtained if each newspaper charged a homogeneous price for its ads.

The cost of producing and distributing newspaper $n \in \{A, B\}$ is $cR_n + kD_n$, where $c \geq 0$ is the cost of each copy sold, $k > 0$ is the cost of each ad published, and R_n and D_n are the circulation and the mass of the ads published, respectively. c measures the cost of printing and distributing the newspaper, and k measures the cost of dealing with the advertiser, as well as the cost of any changes to the blueprint required to incorporate the ad; c and k are exogenously given, whereas R_n and D_n are endogenously determined. To guarantee existence of equilibria with both firms publishing ads, we assume that the cost of advertising each product is not too large, in the sense that $k < \frac{rv(3-a)}{6}$ holds.

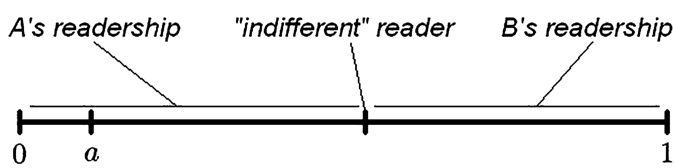

FIGURE 1 An example of an admissible pair of locations of the two newspapers, together with the corresponding circulation of each newspaper, in equilibrium.

Timing and Definition of Equilibrium

The timing is as follows:

1. The newspaper managers set the prices at which the newspapers are offered to the readers.
2. The newspaper managers and the sellers identify the products for which the surplus generated by advertising, net of the advertising cost, is positive, and sign contracts to advertise them.
3. Each reader purchases the newspaper of choice and the products of interest that are advertised there.

As in a number of related papers, such as Anderson and Coate (2005) and Armstrong (2006), we restrict our attention to equilibria in pure strategies. Even approximating the distributions of the endogenous variables, at the equilibria in mixed strategies of the Hotelling model *without* advertising, can be a very demanding task (see Osborne & Pitchik, 1987). The analysis can be expected to be even more complex in the cases with advertising, which are the main objects of the article. Considering also that the set of the parametrizations of the model compatible with equilibria in pure strategies is relatively large, we therefore leave the analysis of possible equilibria in mixed strategies as a topic for future research.

At a Nash equilibrium, the sellers or the newspapers' managers may choose prices lower than the highest prices that the buyers or the readers, respectively, would accept to pay. Such strategies would be justified by the belief that higher prices would be turned down—a belief that would not be tested along the equilibrium path. To rule out this possibility, we restrict our attention to subgame-perfect Nash equilibria (Fudenberg & Tirole, 1991).

For any given pair of prices that the newspapers charge to the readers, $\{p_A, p_B\} \in \mathbb{R}^2$, the circulations (or "readerships") of A and B are, respectively,

$$R_A = \begin{cases} 0, \text{ if } p_A - p_B > t(1-a), \\ \frac{ar}{2}, \text{ if } p_A - p_B = t(1-a), \\ \frac{r}{2}\left(1 + a - \frac{p_A - p_B}{t}\right), \text{ if } p_A - p_B \in (-t(1-a), t(1-a)), \\ r, \text{ if } p_A - p_B \leq -t(1-a), \end{cases} \tag{1}$$

and

$$R_B = r - R_A. \tag{2}$$

The newspapers maximize their profits π_A and π_B. Each seller maximizes her or his profit, which is equal to a fraction $1-\theta$ of the surplus generated by the transactions with the buyers, net of the advertising cost, by choosing a price equal to the buyers' willingness to pay for the respective product, w. The surplus to be divided between the seller and newspaper $n \in \{A, B\}$, $wR_n - k$, is maximized if the products for which $w > q_n$ ($w \leq q_n$) holds are advertised (not advertised), where $q_n \in (0, v]$ is defined by

$$q_n = \begin{cases} v, \text{ if } R_n \leq \dfrac{k}{v}, \\ \dfrac{k}{R_n} \text{ otherwise.} \end{cases} \tag{3}$$

The assumption that the products such that $w = q_n$ are not advertised, although advertising them would also be a part of an optimal strategy for both the newspapers and the sellers, has no impact on the results because the set of such products has measure zero. With the given uniform distribution of w over $[0, v]$, we can conveniently set $m = \frac{\mu}{v}$, and write the measure of the ads published as

$$D_n = \mu \frac{v - q_n}{v} = m(v - q_n).$$

The total surplus generated by the ads, in per-reader terms, is

$$\mu \int_{q_n}^{v} \frac{w}{v} dw = \frac{m\left(v^2 - q_n^2\right)}{2}.$$

Remark 1.

1. *Different values of m, in the next two sections, are understood as different values of the number of potential advertisers, μ, given the highest willingness to pay v.*
2. *The parameters θ and m are multiplied by each other in the equations for the newspapers' profits (Equations 4 and 5). Hence, statements referred to m can be generally interpreted as referred to the product θm.*

The superscript E denotes the equilibrium values of the endogenous variables. In general, an equilibrium is two strategy pairs $\{p_A^E, q_A^E\}$ and $\{p_B^E, q_B^E\}$ allowing each newspaper to maximize profit, if the other newspaper's strategy pair is taken as given. We focus on interior equilibria, at which both A and B have a positive circulation and accept ads, and $\{p_n^E, q_n^E\}$, $n \in \{A, B\}$ are the solutions to

$$\max_{\{p_A, q_A\}} \left\{ \frac{r}{2}\left(1 + a - \frac{p_A - p_B}{t}\right)\left(p_A - c + \frac{\theta m\left(v^2 - q_A^2\right)}{2}\right) - \theta k m (v - q_A) \right\} \tag{4}$$

and

$$\max_{\{p_B, q_B\}} \left\{ \frac{r}{2}\left(1 - a + \frac{p_A - p_B}{t}\right)\left(p_B - c + \frac{\theta m\left(v^2 - q_B^2\right)}{2}\right) - \theta k m (v - q_B) \right\}, \tag{5}$$

respectively:

Definition 1. *An interior equilibrium is two strategy pairs $\{p_A^E, q_A^E\}$ and $\{p_B^E, q_B^E\}$ such that $q_A^E < v$, $q_B^E < v$, $R_A^E \in (0, 1)$ (and, therefore, $R_B^E = 1 - R_A^E \in (0, 1)$), and that if $\{p_B^E, q_B^E\}$ and $\{p_A^E, q_A^E\}$ are respectively taken as given, then:*

1. *The first- and second-order conditions for Equations 4 and 5 are satisfied.*
2. π_A^E *and* π_B^E *are greater than the profits corresponding to any alternative strategy pairs satisfying the first- and second-order conditions for an interior solution to Equations 4 and 5.*
3. π_n^E, $n \in \{A, B\}$ *is greater than the largest profit that newspaper n could realize by publishing no ads.*
4. π_n^E, $n \in \{A, B\}$ *is greater than the largest profit that newspaper n could realize by taking the whole circulation market and publishing ads.*

Requirement 3 in Definition 1 implies $\pi_A^E \geq 0$ and $\pi_B^E \geq 0$; each newspaper can, in fact, always choose a price that makes both its readership and its profit equal to zero.

SOME PRELIMINARY RESULTS

A Benchmark: No Advertisers

If $m = 0$—namely, if there are no sellers willing to advertise—"closed form" expressions for the endogenous variables are available. These expressions provide a convenient benchmark for the analysis of the equilibria that obtain with positive values of m, for which the corresponding closed-form expressions are not available:

Theorem 1. *if* $m = 0$, *there exists a unique equilibrium in pure strategies, at which the prices and the circulations of the newspapers are*

$$p_A^E = c + t\left(1 + \frac{a}{3}\right), \tag{6a}$$

$$p_B^E = c + t\left(1 - \frac{a}{3}\right), \tag{6b}$$

$$R_A^E = \frac{r}{2}\left(1 + \frac{a}{3}\right), \tag{6c}$$

$$R_B^E = \frac{r}{2}\left(1 - \frac{a}{3}\right). \tag{6d}$$

In the hypothetical case of $a = 0$, the two newspapers would be offered to the readers at the same price, and would have circulations of identical size. Positive values of a are associated with higher prices charged by A, allowing B to be the newspaper of choice for a larger fraction of the readers with addresses between a and 1. Essentially, the "captive" readers in $[0, a]$ make identical price reductions relatively less appealing for A because the circulation increases are matched by larger revenue losses from the established readers; an opposite effect makes B's pricing strategy more aggressive. In the language of Fudenberg and Tirole (1984), A's pricing strategy is softened by a "fat-cat effect," whereas B's pricing strategy becomes more aggressive due to a "lean-and-mean" effect. Although these effects run against the exogenous competitive advantage enjoyed by A, by default, A's circulation is indeed larger, in equilibrium.

Larger values of t—namely, a stronger differentiation between the two newspapers, viewed as providers of content—allow the newspapers to charge higher prices and to realize higher profits.

If a is greater than $15 - 6\sqrt{6}$, no equilibria in pure strategies exist: If A chose $p_A = c + t\left(1 + \frac{a}{3}\right)$, B would respond by choosing a price at which the newspaper would be purchased by all readers, rather than $p_B = c + t\left(1 - \frac{a}{3}\right)$. In cases of this type, the model could, in principle, admit equilibria in mixed strategies, which are, however, not considered in this article—see Timing and Definition of Equilibrium section.

A General Characterization of Interior Equilibria

Lemma 1 provides a characterization of the equilibria at which both newspapers have a positive readership. In general, we set $x = p_A - p_B$:

Lemma 1. *An interior equilibrium is characterized by $x = x^E$, implicitly defined by Equation A6 in the Proof of Theorem 2 section of the appendix such that the strategy pairs*

$$\{p_A^E, q_A^E\} = \left\{c + t(1+a) - x^E - \frac{\theta m}{2}\left(v^2 - \left(q_A^E\right)^2\right), \frac{2kt}{r\left(t(1+a) - x^E\right)}\right\}, \tag{7a}$$

$$\{p_B^E, q_B^E\} = \left\{c + t(1-a) + x^E - \frac{\theta m}{2}\left(v^2 - \left(q_B^E\right)^2\right), \frac{2kt}{r\left(t(1-a) + x^E\right)}\right\}, \tag{7b}$$

respectively, dominate any alternative pairs corresponding to interior solutions to Equations 4 and 5, and that Equations A7, A8, A19, and A21 are also satisfied.

To better appreciate the results in Lemma 1, it is useful to briefly consider a case, investigated in an Appendix available from the authors upon request, in which ads are published at a constant cost per reader $g > 0$. In that case, both newspapers would advertise the products for which the readers' willingness to pay is greater than g, regardless of their circulation. Advertising would make the newspapers "more aggressive" in the circulation market, and the prices charged in the benchmark case of $m = 0$, which satisfy $p_A^E = c + t(1+a) - x^E$ and $p_B^E = c + t(1-a) + x^E$, would both be adjusted by an identical term $-\frac{\theta m (v-g)^2}{2}$. Thus, both the difference $p_A^E - p_B^E$ and the circulation of each newspaper would be independent of the number of the potential advertisers.

By contrast, in the case investigated in the article, the expressions for threshold willingness to pay q_A^E and q_B^E, in Equations 7a and 7b, are the ratios between the fixed advertising cost and the circulations. Thus, the marginal benefit of a lower price charged to the readers derives *both* from the higher prices that can be charged to a given set of sellers *and* from the larger number of products that can profitably be advertised. The adjustment term in the equations for p_A and p_B in 7a and 7b equal to $-\frac{\theta m}{2}\left(v^2 - \left(q_n^E\right)^2\right), n \in \{A, B\}$ is now greater, in absolute value, for the newspaper featuring the lower value of q_n^E and, therefore, the greater number of products advertised—which is generally A, as we see in the next section. The quantitative relevance of these effects is positively related to the product θm—the newspapers' participation to the net surplus created by advertising, multiplied by the number of potential advertisers.

Neither c, nor v, appears in Equation A6; thus, x^E is independent of the values of both parameters. Essentially, changes in either parameter have identical effects on both p_A^E and p_B^E, as is evident from Equations 7a and 7b and, therefore, do not affect the difference between the two prices.

EQUILIBRIUM

Existence and Comparative Statics

In this section, we investigate the equilibrium of the model and its comparative statics properties. The main results of the article, concerning existence of equilibrium and the comparative statics with a fixed total circulation, are stated in Theorem 2:

Theorem 2. *For any set of admissible values of the parameters a_0, c_0, k_0, t_0, and v_0, there exists a $\Theta' > 0$ such that if $\theta m < \Theta'$, the following statements hold:*

1. *The model admits a unique interior equilibrium in pure strategies.*
2. *The prices charged to the readers, p_A^E and p_B^E, and the difference $p_A^E - p_B^E$ are both decreasing in m.*
3. *The fractions of the sellers advertising in A and in B, over the total number of the sellers, are respectively increasing and decreasing in m. The profits from advertising, both in absolute terms and in per-reader terms, are increasing in m, for both newspapers, but the change is greater for A.*
4. *A's and B's profits are decreasing in m; the change is greater for B, both in absolute terms and in percentage terms.*

Parts 2 and 3 of Theorem 2 are essentially driven by the effect described earlier: The larger number of sellers willing to advertise makes A more willing than B to lower the price charged to the readers, compared to the benchmark case with no advertising. The competitive effect due to advertising makes differentiation of the content a less-effective shield for the newspapers' profits, as the number of potential advertisers grows. This observation explains the result reported as Part 4 of Theorem 2, which may strike the reader as counterintuitive: The lower profits realized by selling copies of the newspapers to the readers more than offset the increased profits from advertising, and the profits of both newspapers *decrease* if the number of sellers willing to advertise increases. As A's circulation actually increases, A's profit drops by a smaller amount, both in absolute and in relative terms. Two features of this model make this "profit-squeeze" effect especially strong: The aggregate circulation of the newspapers is independent of the prices charged to the readers, and the newspapers are perfect substitutes as gateways to the readers, for the sellers. However, the result is robust in the face of relatively small violations of these assumptions and, in any case, the effect behind it can be relevant even in the face of more substantial violations of the assumptions.

A routine written in *wxMaxima*, available from the authors upon request, allows us to investigate cases with arbitrarily large values of m by numerical experiments. In Figure 2, we report the results for a set of examples, featuring $a = \frac{1}{10}, c = \frac{1}{2}, k = \frac{1}{2}, r = 1, t =$

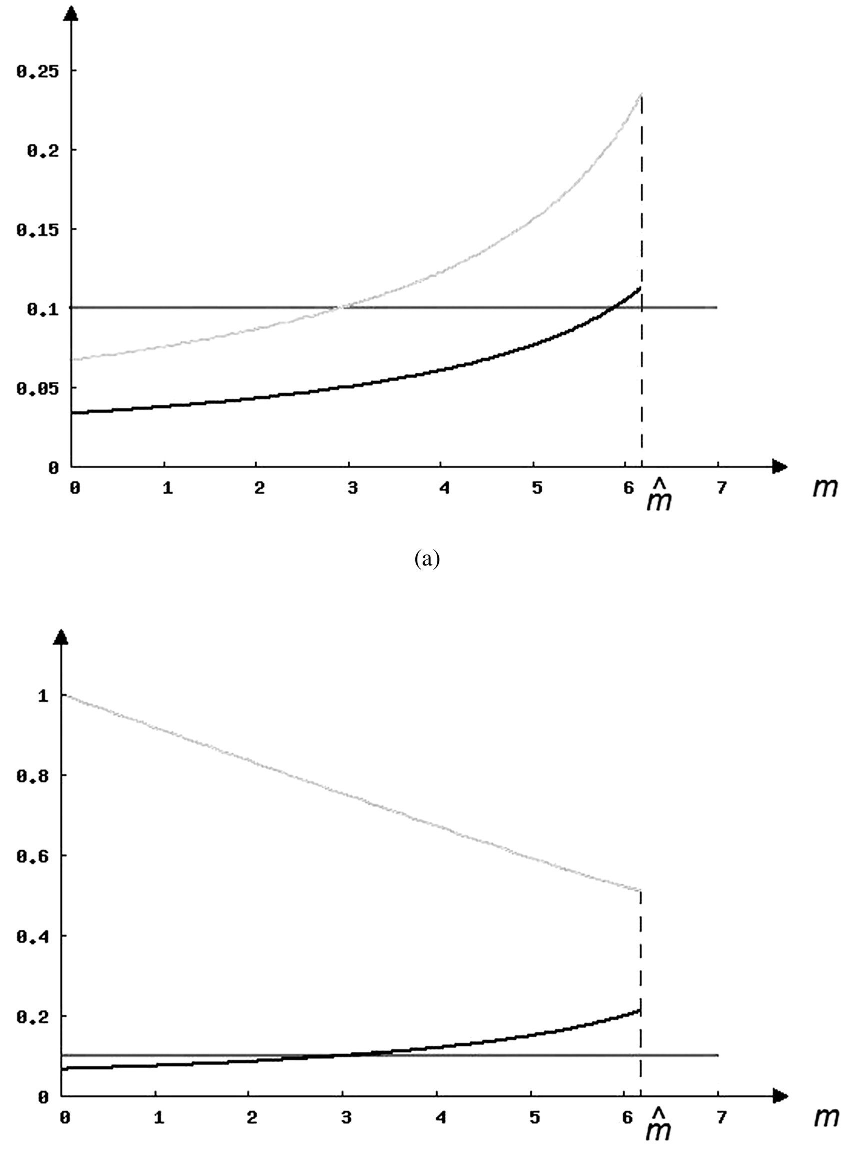

FIGURE 2 (a) The ratios $\frac{R_A^E - R_B^E}{R_A^E + R_B^E}$ (black curve) and $\frac{D_A^E - D_B^E}{D_A^E + D_B^E}$ (light gray curve), for different values of m, plotted together with a (dark gray curve). (b) The ratios $\frac{\pi_A^E - \pi_B^E}{\pi_A^E + \pi_B^E}$ (black curve) and $\frac{\pi_A^E + \pi_B^E}{\bar{\pi}_A^E + \bar{\pi}_B^E}$ (light gray curve), where $\bar{\pi}_A^E$ and $\bar{\pi}_B^E$ are the newspapers' profits when $m = 0$, for different values of m, plotted together with a (dark gray curve).

$2, \theta = \frac{1}{3}$, and $v = \frac{3}{2}$, and values of m ranging between 0 and $\hat{m} = 5.7678$. $\hat{m}$ is a numerical approximation of the highest value of m compatible with existence of a pure strategy equilibrium, at which both newspapers advertise some products, given the values of the remaining parameters.

Figure 2a reports the difference between the circulations and the difference between the advertising revenues of A and B. Figure 2b reports both the difference between the profits and the sum of the profits of A and B. To highlight the role of the endogenous effects, we report the differences between variables referred to A and variables referred to B as fractions of the sums of the same variables—namely, we actually report the ratios $\frac{R_A^E - R_B^E}{R_A^E + R_B^E}, \frac{D_A^E - D_B^E}{D_A^E + D_B^E}$ and $\frac{\pi_A^E - \pi_B^E}{\pi_A^E + \pi_B^E}$—and we include the parameter a, measuring A's exogenous competitive advantage, in both parts of the figure. In Figure 2b, to highlight the effects of a greater number of potential advertisers, we also report the sum of the profits as a fraction of the value of the profits when $m = 0$, $\bar{\pi}_A^E$ and $\bar{\pi}_B^E$—namely, we actually report the ratio $\frac{\pi_A^E + \pi_B^E}{\bar{\pi}_A^E + \bar{\pi}_B^E}$.

With values of m greater than $\hat{m}$, B would respond to A's candidate equilibrium strategy by charging relatively high prices to the readers in a small market niche and exiting the advertising market. This response would invite a more aggressive pricing strategy by A, which would, in turn, boost the discontinuous jump in the gap between the prices and the circulations of the two newspapers; the fully interior, pure strategy equilibrium would thus unravel. An alternative possibility, identified in the numerical experiments, is qualitatively more similar to the unraveling of the equilibrium in the case with no advertisers, investigated earlier: $\hat{m}$ can be such that, for higher values of m, B would respond to A's candidate equilibrium strategy by charging the highest price allowing to take the whole circulation market and by publishing the largest profitable number of ads. This would be the case, for example, if a were increased from $\frac{1}{10}$ to $\frac{1}{4}$, without changing the values of the remaining parameters; the estimated critical value of m, in this case, is $\hat{m} = 4.5677$. In both cases, no other values of x, defined by Equation A6 were found, for $m \leq \hat{m}$.

A non-negativity constraint on the prices charged to the readers can generally hinder the price reductions driven by increases in m and thereby dampen the tendency toward a lower profitability of the newspapers. Numerical experiments reveal that negative prices require relatively large values of θ, m, and v and relatively small values of k; in such circumstances, the opportunity to advertise a "large" set of products to larger numbers of readers creates a strong downward pressure on p_A and p_B. Low values of the printing cost c and the differentiation parameter t also make negative prices relatively more likely. If the constraint were only binding for one of the newspapers, the strategic position of such newspaper would necessarily be weakened. From this point of view, cases featuring relatively small values of m tend to resemble the case of $m = 0$ in that B charges a lower price to the readers and may, therefore, be the only newspaper facing the constraint. The tendency may be reversed for relatively high values of m. In any case, negative prices do not appear to be a "typical" feature of the model.

The results in Theorem 3 concern the responses to changes in the aggregate circulation, r:

Theorem 3. *For any set of admissible values of the parameters a_0, c_0, k_0, r_0, t_0, and v_0, there exists a $\Theta'' > 0$ such that if $\theta m < \Theta''$, the following statements hold:*

1. *The prices charged to the readers, p_A^E and p_B^E, are both decreasing in r while the difference $p_A^E - p_B^E$ is increasing in r.*
2. *The fractions of the sellers advertising in A and in B, over the total number of the sellers, are both increasing in r. The profits from advertising, both in absolute and in per-reader terms, increase. The former change is greater in the case of A, whereas the latter change is greater in the case of B.*
3. *The newspapers' profits π_A^E and π_B^E are increasing in r; the change is greater in the case of A.*

In the case of increases in r, unlike in the case of increases in m, $p_A - p_B$ increases, the fraction of the sellers advertising in newspaper B also increases, and the profits of the two newspapers increase. The increase in the profit from advertising, in per-reader terms, is greater for B. Given any two prices prices charged to the readers, and the circulations corresponding to them, higher values of r increase:

1. The revenues realized by selling the newspapers to the readers.
2. The revenues realized by advertising any given product.
3. The number of products that can profitably be advertised.

Only the effect in 3 has a counterpart in the case of increases in m. As in that case—and together, now, with the effect in 2—this effect makes price reductions relatively more appealing for the newspapers, whereas the effect in 1 runs in the opposite direction. The net result is a relatively mild increase in the competition for readers, which leaves room for an increase in the newspapers' profits; the greater circulation makes A relatively less eager than B to cut the price charged to the readers.

The results in Theorem 3 are of particular interest in the light of the trends currently observed in most newspaper markets, where advertisers are shifting from newspapers to other platforms—most notably, online venues—and the newspapers' profits are falling; see, for example, *The Economist* (2009). In the benchmark model with an advertising cost proportional to the circulation, mentioned in the discussion of Lemma 1, a shrinking total circulation due, for example, to a switch to online readership, would be associated with lower revenues from the circulation, lower revenues per product advertised, and lower profits. The same effects would also emerge in our model where, additionally, the threshold willingness to pay below which products are not advertised would increase. Thus, not only the total profits from advertising, but also the profit from advertising in per-reader terms would decrease. Our model would also predict a shrinking difference between the profits of the two newspapers and higher prices charged to the readers while a result qualitatively similar to the former one would also emerge with an advertising cost proportional to the circulation—the same is not true for the latter result.

Welfare

The (standard) measure of welfare that we consider is the total surplus generated in the newspapers market, both directly through the newspapers' content and indirectly through the ads published. As we mentioned earlier, with prices of the ads determined by bargaining between the newspaper manager and each seller, each newspaper will publish the surplus-maximizing number of ads, *given* its circulation. Only values of the difference $p_A^E - p_B^E$ determining a "wrong" partition of the aggregate circulation can, therefore, stand in the way of surplus maximization. As we know from Theorem 2, $p_A^E > p_B^E$ holds if $m = 0$—namely, the watershed between the readers of the two newspapers is closer to A than to B. As each readers' preferences are symmetric, it is then evident that if there are no advertisers, the price differential necessarily works against welfare maximization—namely, that A will have "too few readers." It would seem reasonable to conjecture that this situation will persist, at least for values of m that are "small," relatively to r. Theorem 4 states that this conjecture is correct:

Theorem 4.

1. *For any set of admissible values of the parameters a_0, c_0, k_0, r_0, t_0, and v_0, there exists a $\Theta^W > 0$ such that if $\theta m < \Theta^W$, the difference between the readerships, $R_A^E - R_B^E$, is smaller than the difference that would maximize the total surplus; the gap between the equilibrium and the optimal level of $R_A - R_B$ is increasing in m.*
2. *For any set of admissible values of the parameters a_0, c_0, k_0, r_0, t_0, and v_0, and conveniently small values of θ_0 and m_0, there exists an open neighborhood of r_0, $\mathcal{B}(r_0)$, such that if $r \in \mathcal{B}(r_0)$, the gap between the equilibrium and the optimal level of $R_A - R_B$ is increasing in r.*

A hypothetical planner, maximizing the total surplus, would respond to an increase in the number of potential advertisers by boosting the circulation gap between the two newspapers, to better exploit the economies of scale in advertising. Conversely, the planner would respond to an increase in the aggregate circulation by reducing the circulation gap, due to the greater importance of the utility deriving from the newspapers' content. In both cases, the changes respectively considered increase the gap between the equilibrium and the surplus-maximizing partition of the readers between the two newspapers.

CONCLUSION

We have investigated a model where two newspapers compete between them for readers and advertisers. The readers feature heterogeneous preferences for the newspapers, and each newspaper can publish additional ads at a cost that is independent of its circulation. The model can account for the empirical regularity that the revenues and the profits from advertising of the newspapers with a larger circulation are disproportionately larger than those of their competitors. Furthermore, larger numbers of potential advertisers translate into lower profits, for both newspapers. The model can also account for a simultaneous drop in the newspapers' aggregate circulation, the ads published, and the newspapers' profitability of the type currently observed in many newspaper markets.

As in a number of related works on two-sided platforms, we have assumed fixed locations of the newspapers in the space of the readers' preferences. One topic for future research is the investigation of a version of the model in which the newspapers can choose their locations. On a related note, we could allow additional newspapers to enter the market, as far as the profit from the operations are sufficient to cover the entry cost. The assumptions concerning the readers' preferences could also be relaxed. In particular, it would be interesting to allow the readers to purchase more than one newspaper or to investigate the consequences of vertical differentiation of both the newspapers and the sellers' products.

In principle, the results of the article could have a bearing on other two-sided platforms that provide both information or entertainment content and advertising, besides newspapers, such as radio, movies, television, Internet sites, and search engines, as far as the cost of advertising each product increases less than proportionally with the circulation. Any such extension of the article should, however, carefully consider some platform-specific features, such as the possibility for the users to limit their exposure to the ads that are not of interest to them. This possibility can be relatively large in the case of Internet sites and search engines and much smaller in the case of radio, movies, and television.

ACKNOWLEDGMENTS

Financial support from the Hamrin Foundation is gratefully acknowledged. We also thank Brendan Cunningham (the Co-Editor), an anonymous referee, Anthony Creane, Charlotta Mellander, and the participants in the Conference of the Applied Econometrics Association on "Media and Communication," held in Paris, November 2007, for their comments.

REFERENCES

Anderson, S. P., & Coate, S. (2005). Market provision of broadcasting: A welfare analysis. *Review of Economic Studies, 72,* 947–972.

Anderson, S. P., & Gabszewicz, J. J. (2006). The media and advertising: A tale of two-sided markets. In V. Ginsburgh & D. Throsby (Eds.), *Handbook of the economics of art and culture* (pp. 567–614). London: Elsevier.

Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics, 37,* 668–691.

Blair, R. D., & Romano, R. E. (1993). Pricing decisions of the newspaper monopolist. *Southern Economic Journal, 59,* 721–732.

Brody, J. H., & Picard, R. G. (1997). *The newspaper publishing industry.* Boston: Allyn & Bacon.

Corden, W. M. (1953). The maximisation of profit by a newspaper. *Review of Economic Studies, 20,* 181–190.

Cover, J. H., Thompson, P. J., & Cohenour, V. J. (1931). Newspaper advertising rates and circulation. *Journal of Business of the University of Chicago, 4,* 115–126.

The Economist. (2009, May 16). The news business: Tossed by a gale. *391*(8631), 73–75.

Ferrando, J., Gabszewicz, J. J., Laussel, D., & Sonnac, N. (2008). Intermarket network externalities and competition: An application to the media industry. *International Journal of Economic Theory, 4,* 357–379.

Fudenberg, D., & Tirole, J. (1984). The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. *American Economic Review, 74,* 361–366.

Fudenberg, D., & Tirole, J. (1991). *Game theory.* Cambridge, MA: MIT Press.

Furhoff, L. (1973). Some reflections on newspaper concentration. *Scandinavian Economic History Review, 21,* 1–27.

Gabszewicz, J. J., Garella, P., & Sonnac, N. (2007). Newspapers' market shares and the theory of the circulation spiral. *Information Economics and Policy, 19,* 405–413.

Gabszewicz, J. J., Laussel, D., & Sonnac, N. (2001). Press advertising and the ascent of the "Penseè unique." *European Economic Review, 45,* 641–651.

Gustafsson, K. E. (1978). The circulation spiral and the principle of household coverage. *Scandinavian Economic History Review, 26,* 1–14.

Inland Press Association & International Newspaper Financial Executives. (2008). *Inland/INFE national cost and revenue study.* Des Plaines, IL: Inland Press Association/Inland Press Foundation.

Kaiser, U. (2006). *Magazine readers: Ad-lovers, ad-haters or ad-neutrals?* (Discussion Paper No. 2006-09). Centre for Economic and Business Research, Copenhagen Business School.

Laffont, J. J., Rey, P., & Tirole, J. (1998a). Network competition: I. Overview and nondiscriminatory pricing. *RAND Journal of Economics, 29,* 1–37.

Laffont, J. J., Rey, P., & Tirole, J. (1998b). Network competition: II. Price discrimination. *RAND Journal of Economics, 29,* 38–56.

Osborne, M. J., & Pitchik, C. (1987). Equilibrium in Hotelling's model of spatial competition. *Econometrica, 55,* 911–922.

Picard, R. (1998). A note on the relations between circulation size and newspaper advertising rates. *Journal of Media Economics, 11*(2), 47–55.

PricewaterhouseCoopers. (2007). *Spending on convergent platforms will exceed 50% of global entertainment & media spending by 2011.* Retrieved November 4, 2008, from http://www.pwc.com

Reddaway, W. (1963). The economics of newspapers. *Economic Journal, 73,* 201–218.

Resende, J. (2008). The economic advantage of being the "voice of the majority." *Journal of Media Economics, 21,* 158–190.

Tirole, J., & Rochet, J.-C. (2006). Two-sided markets: A progress report. *RAND Journal of Economics, 37,* 645–667.

Thompson, R. S. (1989). Circulation versus advertiser appeal in the newspaper industry: An empirical investigation. *Journal of Industrial Economics, 37,* 259–271.

APPENDIX

Proof of Theorem 1

By setting $m = 0$ in Equations 4 and 5, and differentiating the same equations with respect to p_A and to p_B, we obtain the system of first-order conditions:

$$\begin{cases} \dfrac{\partial \pi_A}{\partial p_A} = \dfrac{r}{2t}\left(t(1+a) + c - 2p_A + p_B\right) = 0, \\ \dfrac{\partial \pi_B}{\partial p_B} = \dfrac{r}{2t}\left(t(1-a) + c + p_A - 2p_B\right) = 0, \end{cases} \tag{A1}$$

and the second-order conditions:

$$\frac{\partial^2 \pi_A}{\partial p_A^2} = \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{r}{t}, \tag{A2}$$

which are always verified. The prices p_A^E and p_B^E in Equations 6a and 6b—which define the strictly positive circulations R_A^E and R_B^E in Equations 6c and 6c, via Equations 1 and 2—are the unique solution to A1 and, therefore, satisfy Requirement 1, in Definition 1. Requirement 2 is satisfied because each equation in A1 defines a unique response for the respective newspaper, given the price chosen by the other newspaper. Requirement 3 is satisfied because the profits

$$\pi_A^E = \frac{rt(3+a)^2}{18} \tag{A3}$$

and

$$\pi_B^E = \frac{rt(3-a)^2}{18} \tag{A4}$$

are both positive, and with $m = 0$, p_A^E and p_B^E are already optimal prices conditional on no ads being published. Relatively to Requirement 4, we can use $p_A^E - p_B^E = x^E = \frac{2ta}{3}$ and the equations for the profits that the newspapers could realize by taking the whole circulation market—which coincide with A20 and A22 with m set equal to 0—to write $\pi_A^E \geq \pi_A^T$ and $\pi_B^E \geq \pi_B^T$ as $\frac{r(t(1+a)-x)^2}{2t} \geq rx$ and as $\frac{r(t(1-a)+x)^2}{2t} \geq r(2at-x)$. These conditions require $a \neq 3$ and $a \notin \left(15-6\sqrt{6}, 15+6\sqrt{6}\right)$, respectively, and are, therefore, satisfied under the maintained assumption that $a \in \left(0, 15-6\sqrt{6}\right)$.

Proof of Lemma 1

Differentiation of the maximands in Equations 4 and 5 yields the first-order conditions for the two newspapers, at an interior equilibrium:

$$\begin{cases} \dfrac{\partial \pi_A}{\partial p_A} = \dfrac{r}{2t}\left(t(1+a)+c-2p_A+p_B-\dfrac{\theta m}{2}\left(v^2-q_A^2\right)\right) = 0, \\ \dfrac{\partial \pi_A}{\partial q_A} = \theta m \left(k - \dfrac{r\left(t(1+a)-p_A+p_B\right)q_A}{2t}\right) = 0, \\ \dfrac{\partial \pi_B}{\partial p_B} = \dfrac{r}{2t}\left(t(1-a)+c+p_A-2p_B-\dfrac{\theta m}{2}\left(v^2-q_B^2\right)\right) = 0, \\ \dfrac{\partial \pi_B}{\partial q_B} = \theta m \left(k - \dfrac{r\left(t(1-a)+p_A-p_B\right)q_B}{2t}\right) = 0. \end{cases} \tag{A5}$$

From these expressions, we obtain the expressions in Equation 7 by setting $x = p_A - p_B$, and by using x^E to denote the equilibrium values of x. By using the expressions for p_A and p_B in 7a and 7b to express the difference $p_A - p_B$, and rearranging, we then obtain

$$2at - 3x - \frac{8\theta k^2 m t^3 (at-x)}{r^2(t(1+a)-x)^2(t(1-a)+x)^2} = 0. \tag{A6}$$

$x^E \in (-t(1-a), t(1-a))$ and $x^E \in \left(-t\left(1-a-\frac{2k}{rv}\right), t\left(1+a-\frac{2k}{rv}\right)\right)$ are required for both newspapers to have a positive circulation and to accept a positive number of ads. Notice that $-t(1-a) < -t\left(1-a-\frac{2k}{rv}\right)$ certainly holds, whereas $t(1-a)$ may be either larger or smaller than $t\left(1+a-\frac{2k}{rv}\right)$, compatibly with the given parameter restrictions. The conditions for an interior equilibrium are then summarized by

$$x^E \in \left(-t\left(1-a-\frac{2k}{rv}\right), \min\left\{t(1-a), t\left(1+a-\frac{2k}{rv}\right)\right\}\right). \tag{A7}$$

We then proceed to examining the restrictions deriving from Requirements 1 through 4, in Definition 1.

Requirement 1. With $\theta m > 0$, the second-order conditions (Equation A2) are complemented by

$$\left(\frac{\partial^2 \pi_A}{\partial p_A^2}\frac{\partial^2 \pi_A}{\partial q_A^2} - \left(\frac{\partial^2 \pi_A}{\partial p_A \partial q_A}\right)^2\right)\Bigg|_{p_A - p_B = x^E, q_A = q_A^E}$$

$$= \theta m\left(\frac{r^2\left(1+a-\frac{x}{t}\right)}{2t} - \frac{\theta k^2 m}{(t(1+a)-x)^2}\right) > 0$$

and by

$$\left(\frac{\partial^2 \pi_B}{\partial p_B^2}\frac{\partial^2 \pi_B}{\partial q_B^2} - \left(\frac{\partial^2 \pi_B}{\partial p_B \partial q_B}\right)^2\right)\Bigg|_{p_A - p_B = x^E, q_B = q_B^E}$$

$$= \theta m\left(\frac{r^2\left(1-a+\frac{x}{t}\right)}{2t} - \frac{\theta k^2 m}{(t(1-a)+x)^2}\right) > 0,$$

respectively, for A and for B. Using the fact that $t(1-a)+x^E > 0$ and $t(1+a)-x^E > 0$ both hold at an interior equilibrium, we can summarize the additional requirements as

$$x^E \in \left(\sqrt[3]{\frac{2\theta m k^2 t^2}{r^2}} - t(1-a), \sqrt[3]{\frac{2\theta m k^2 t^2}{r^2}} + t(1+a)\right). \tag{A8}$$

Requirement 2. The qualification that the two strategy pairs must dominate any alternative strategies corresponding to interior solutions to Equations 4 and 5 is necessary because the values of x^E defined by A6 identify the only *reciprocally consistent* values of p_A and p_B that, together with the values of q_A and q_B associated with them, satisfy the first-order conditions of the two newspapers. However, for any value of p_B (p_A), the "reduced form" first-order conditions that can be obtained from A5

$$t(1+a)+c-2p_A+p_B-\frac{\theta m}{2}\left(v^2-\left(\frac{2kt}{r\left(t(1+a)-p_A+p_B\right)}\right)^2\right)=0, \tag{A9}$$

$$t(1-a)+c+p_A-2p_B-\frac{\theta m}{2}\left(v^2-\left(\frac{2kt}{r\left(t(1-a)+p_A-p_B\right)}\right)^2\right)=0, \tag{A10}$$

respectively viewed as functions of p_A (p_B), for any given value of p_B (p_A), are generically solved by multiple values of p_A and p_B—not necessarily all associated with an interior solution.

Requirement 3. To identify the optimal price that A would charge to the readers, *conditional* on no ads being published, let π_A^γ denote the profit that A can derive by publishing no ads if $p_B = p_B^E$. We have

$$\frac{\partial \pi_A^\gamma}{\partial p_A} = \frac{\partial\left((p_A - c)\frac{r}{2t}\left(t(1+a) - p_A + p_B^E\right)\right)}{\partial p_A}$$

$$= \frac{r}{2t}\left(c + t(1+a) - 2p_A + p_B^E\right) = 0,$$

which is verified if $p_A = p_A^\emptyset$, where $p_A^\emptyset = \frac{c+t(1+a)+p_B^E}{2}$. The second-order condition $\frac{\partial^2 \pi_A^\gamma}{\partial p_A^2} = -\frac{r}{t} < 0$ is always verified; the same holds for $\frac{\partial^2 \pi_B^\gamma}{\partial p_B^2}$. As $p_A^\emptyset$ could, in principle, take on negative values, if p_B^E were substantially smaller than 0, a necessary equilibrium condition is then

$$\pi_A^E \geq \max\{0, \pi_A^\emptyset\}, \tag{A11}$$

where

$$\pi_A^E = \frac{1}{2t}\left(r(t(1+a) - x^E)^2 - \frac{\theta m\left(r^2v^2\left(t(1+a) - x^E\right)^2 - 4k^2t^2\right)}{2r\left(t(1+a) - x^E\right)}\right) + \Phi_A^E, \tag{A12}$$

$$\Phi_A^E = \frac{\theta m(rv\left(t(1+a) - x^E\right) - 2kt)^2}{4rt\left(t(1+a) - x^E\right)}, \tag{A13}$$

and

$$\pi_A^\emptyset = \frac{r}{32t}\left(\theta m\left(v^2 - \left(\frac{2kt}{r(t(1-a) + x^E)}\right)^2\right) - 2(2t + x^E)\right)^2 \tag{A14}$$

is the value of π_A^γ when $p_A = p_A^\emptyset$.

Similarly, if $p_A = p_A^E$, we have

$$\frac{\partial \pi_B^\gamma}{\partial p_B} = \frac{\partial\left((p_B - c)\frac{r}{2t}\left(t(1-a) + p_A^E - p_B\right)\right)}{\partial p_B}$$

$$= \frac{r}{2t}\left(c + t(1-a) + p_A^E - 2p_B\right) = 0,$$

which is satisfied if $p_B = p_B^{\emptyset}$, where $p_B^{\emptyset} = \frac{c+t+(1-a)+p_A^E}{2}$. A further necessary condition for an equilibrium is then

$$\pi_B^E \geq \max\{0, \pi_B^{\emptyset}\}, \tag{A15}$$

where

$$\pi_B^E = \frac{1}{2t}\left(r\left(t(1-a)+x^E\right)^2 - \frac{\theta m\left(r^2 v^2\left(t(1-a)+x^E\right)^2 - 4k^2t^2\right)}{2r\left(t(1-a)+x^E\right)}\right) + \Phi_B^E, \tag{A16}$$

$$\Phi_B^E = \frac{\theta m\left(rv\left(t(1-a)+x^E\right)-2kt\right)^2}{4rt\left(t(1-a)+x^E\right)}, \tag{A17}$$

$$\pi_B^{\emptyset} = \frac{r}{32t}\left(\theta m\left(v^2 - \left(\frac{2kt}{r\left(t(1+a)-x^E\right)}\right)^2\right) - 2(2t-x^E)\right)^2. \tag{A18}$$

Requirement 4. We use the superscript T to denote variables referred to a hypothetical response at which a newspaper chooses the highest price allowing it to be chosen by all readers, given the other newspaper's candidate equilibrium strategy. In such conditions, the advertising strategy features $q_n^T = \frac{k}{r}, n \in \{A, B\}$. The price that would allow A to be preferred by all readers, defined by $R_A = 1$, is $p_A^T = p_B^E - t(1-a)$. Considering A11, some algebra reveals that

$$\pi_A^E \geq \max\{0, \pi_A^{\emptyset}, \pi_A^T\}, \tag{A19}$$

where

$$\pi_A^T = rx^E + \frac{\theta m}{2r}\left(\left(\frac{2kt}{t(1-a)+x^E}\right)^2 - k(2rv-k)\right), \tag{A20}$$

is a necessary condition for equilibrium. Furthermore, any price $p < p_B^T$, where p_B^T is defined by $R_B = 1-a$, would allow B to be preferred by all readers. Notice that if the reader with address $1-a$ strictly prefers B to A, then all readers in $[0, a]$ strictly prefer B to A. Also, if $p = p_B^T$, the readers in $[0, a]$ could optimally choose A, in the face of a deviation; thus, the strict inequality $p < p_B^T$ is necessary. As $p_B^T = p_A^E - t(1-a)$, considering A15, a further necessary condition for equilibrium is then

$$\pi_B^E \geq \max\{0, \pi_B^{\emptyset}, \pi_B^T\}, \tag{A21}$$

where

$$\pi_B^T = r\left(2at - x^E\right) + \frac{\theta m}{2r}\left(\left(\frac{2kt}{t(1+a)-x^E}\right)^2 - k(2rv-k)\right). \tag{A22}$$

Proof of Theorem 2

Part 1. As we know from the proof of Theorem 1, if $m = 0$, (A6) is solved by $x = \frac{2at}{3}$. Furthermore, we have

$$\left.\frac{dx}{dm}\right|_{m=0,x=\frac{2at}{3}} = -\left.\frac{\dfrac{\partial\left(2at - 3x - \dfrac{8\theta k^2 m t^3 (at - x)}{r^2(t(1+a)-x)^2(t(1-a)+x)^2}\right)}{\partial m}}{\dfrac{\partial\left(2at - 3x - \dfrac{8\theta k^2 m t^3 (at - x)}{r^2(t(1+a)-x)^2(t(1-a)+x)^2}\right)}{\partial x}}\right|_{m=0,x=\frac{2at}{3}}$$

$$= -\frac{72\theta a k^2}{r^2(3+a)^2(3-a)^2} < 0. \tag{A23}$$

(Observing that $\frac{\partial\left(2at-3x-\frac{8\theta k^2 m t^3(at-x)}{r^2(t(1+a)-x)^2(t(1-a)+x)^2}\right)}{\partial x} = -3 \neq 0$ would suffice, for our purposes, but the inequality in A23 is useful in the proof of Part 2). The Implicit Function Theorem guarantees the existence of a continuously differentiable, decreasing function X_m, defined over a conveniently small right neighborhood of $m = 0$, such that A6 holds if $x = X_m$. $k < \frac{rv(3-a)}{6}$ guarantees that at $m = 0$, we have

$$X_m = \frac{2at}{3} \in \left(-t\left(1 - a - \frac{2k}{rv}\right), \min\left\{t(1-a), t\left(1 + a - \frac{2k}{rv}\right)\right\}\right);$$

hence, continuity guarantees that X_m is compatible with interior values of the strategic variables, for suitably small values of m.

The "reduced form" first-order conditions A9 and A10, in the proof of Lemma 1, cannot be satisfied by values of p_A and p_B such that the differences $p_A - p_B^E$ and $p_A^E - p_B$ take on values in the interval $(-t(1-a), t(1-a))$, other than p_A^E and p_B^E, at least for conveniently small values of m. Hence, the interior best responses to the candidate equilibrium strategies are unique, for suitably small values of m.

The assumption that $k < \frac{rv(3-a)}{6}$ guarantees that A7 is satisfied if $x = X_m$, evaluated at $m = 0$. Continuity of X_m guarantees then that A7 will also be satisfied if $x = X_m$, at least for sufficiently small, positive values of m. Similarly, at $m = 0$, A8 reduces to $\frac{2at}{3} \in (-t(1-a), t(1+a))$. As $t > 0$, this condition can be reformulated as $a \in (-3, 3)$, and is, therefore, certainly satisfied. Continuity of X_m, together with continuity in m of the bounds in A8, implies that the second-order conditions are also necessarily verified for conveniently small values of m. Furthermore, as we know from A3 and A4, both π_A^E and π_B^E are strictly positive at $m = 0$. Hence, by continuity, both variables must also be strictly positive for conveniently small values of m. As $p_A^{\emptyset}$ and $p_B^{\emptyset}$ maximize the profits if no ads are published, both $p_A^E = p_A^{\emptyset}$

and $p_B^E = p_B^\emptyset$, and therefore $\pi_A^E = \pi_A^\emptyset$ and$\pi_B^E = \pi_B^\emptyset$, hold at $m = 0$. We also have

$$\left(\frac{d\pi_A^E}{dm} - \frac{d\pi_A^\emptyset}{dm}\right)\bigg|_{m=0,x=\frac{2at}{3}} = \frac{\theta(rv(3+a)-6k)^2}{12r(3+a)} > 0,$$

$$\left(\frac{d\pi_B^E}{dm} - \frac{d\pi_B^\emptyset}{dm}\right)\bigg|_{m=0,x=\frac{2at}{3}} = \frac{\theta(rv(3-a)-6k)^2}{12r(3-a)} > 0.$$

Thus, both $\pi_A^E > \pi_A^\emptyset$ and $\pi_B^E > \pi_B^\emptyset$ hold for values of m in a right-neighborhood of 0. $\pi_A^E > \pi_A^T$ and $\pi_B^E > \pi_B^T$ certainly hold if $m = 0$, by uniqueness of the optimal values of p_A and p_B. Furthermore, continuity of A12, A16, A20, and A22 guarantees that the inequalities $\pi_A^E > \pi_A^T$ and $\pi_B^E > \pi_B^T$ must still hold for conveniently small values of m.

To establish (local) uniqueness of the interior equilibrium associated with X_m, we must confront the fact that A6 is effectively a polynomial equation in x of degree 5; while $X_m = \frac{2at}{3}$ is the unique real solution, if $m = 0$, four additional real solutions exist, at least for values of m in a small right-neighborhood of 0. Now,

$$-\frac{\dfrac{\partial\left(2at - 3x - \dfrac{8\theta k^2 m t^3(at-x)}{r^2(t(1+a)-x)^2(t(1-a)+x)^2}\right)}{\partial m}}{\dfrac{\partial\left(2at - 3x - \dfrac{8\theta k^2 m t^3(at-x)}{r^2(t(1+a)-x)^2(t(1-a)+x)^2}\right)}{\partial x}} \neq 0$$

holds if $x \in (\mathcal{B}'(-t(1-a))\backslash -t(1-a))$ and if $x \in (\mathcal{B}''(t(1+a))\backslash(t(1+a)))$, where $\mathcal{B}'(-t(1-a))$ and $\mathcal{B}''(t(1+a))$ are conveniently small neighborhoods of $-t(1-a)$ and of $t(1+a)$; the numerator of the left-hand side (LHS) of the previous inequality only vanishes at $-t(1-a)$ and at $t(1+a)$, over the given neighborhoods, while the denominator converges in absolute value to $32\theta k^2 m t^5 > 0$, if x converges to either $-t(1-a)$, or $t(1+a)$. It is then evident that, at least if we consider relatively small values of m, the Implicit Function Theorem guarantees existence of four distinct, continuous functions X_i, $i \in \{1, 2, 3, 4\}$, besides X_m, such that A6 holds when $x = X_i$, and that we can choose indexes so that $\lim_{m\downarrow 0}\{X_1\} = \lim_{m\downarrow 0}\{X_2\} = -t(1-a)$ and $\lim_{m\downarrow 0}\{X_3\} = \lim_{m\downarrow 0}\{X_4\} = t(1+a)$ both hold; the procedure is illustrated by Figure A1. Local nonexistence of interior equilibria, besides the one associated with X_m, is then established by observing that

$$X_i \notin \left(-t\left(1 - a - \frac{2k}{rv}\right), \min\left\{t(1-a), t\left(1 + a - \frac{2k}{rv}\right)\right\}\right)$$

must hold for values of m in a suitably small right neighborhood of 0, for any $i \in \{1, 2, 3, 4\}$: Any solution of A6, besides X_m, is certainly incompatible with the condition that the circulation of each newspaper must be sufficiently large to justify a positive volume of advertising, and possibly even with positivity of the circulation, for one of the newspapers.

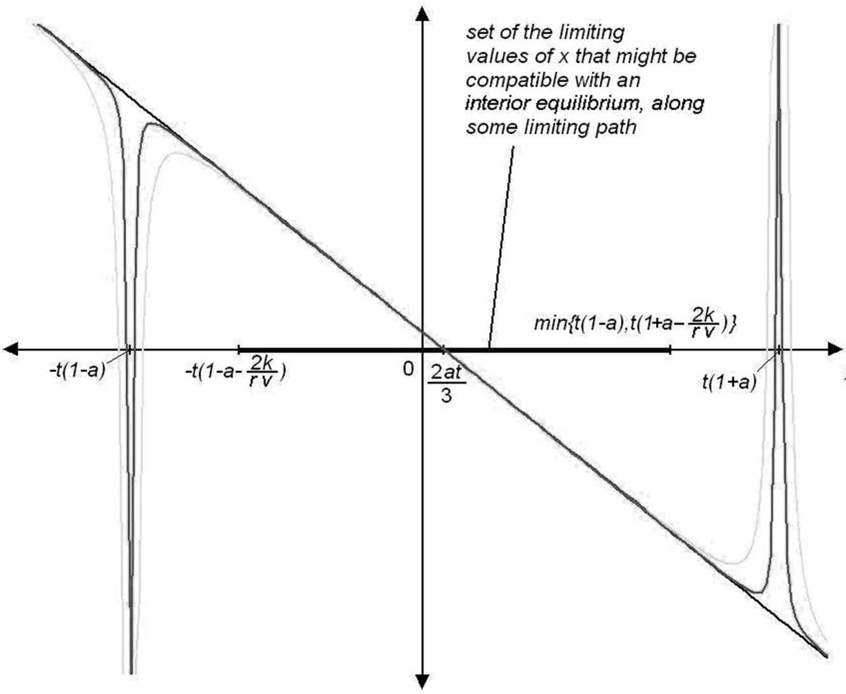

FIGURE A1 A plot of the LHS of Equation A6, viewed as a function of x, for different values of m, given the values of the remaining parameters. Curves plotted in lighter shades of gray colors correspond to higher values of m.

Part 2. The proofs of this part of the theorem and the ones that follow are based on the signs of the relevant derivatives, evaluated at $m = 0, x = \frac{2at}{3}$, and on continuity of the derivatives, implying that their sign remains unchanged, at least for suitably small values of m. The expression for $\frac{dX_m}{dm}$ in A23, whose sign immediately allows to establish that $p_A^E - p_B^E$ is decreasing in m, are repeatedly used to evaluate the derivatives. Using $k < \frac{rv(3-a)}{6}$, we can write

$$\left.\frac{dp_A^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial p_A^E}{\partial m} + \frac{\partial p_A^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = -\frac{\theta}{2}\left(v^2 - \frac{36k^2\left(a^2-2a+9\right)}{r^2(3-a)^2(3+a)^2}\right) < 0,$$

$$\left.\frac{dp_B^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial p_B^E}{\partial m} + \frac{\partial p_B^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = -\frac{\theta}{2}\left(v^2 - \frac{36k^2\left(a^2+2a+9\right)}{r^2(3-a)^2(3+a)^2}\right) < 0.$$

Part 3. The fractions of the sellers advertising in the two newspapers are positively related to the levels of the threshold willingness to pay, above which the products are advertised in each newspaper. The statement concerning the effects of changes in m on them follows then immediately from the result concerning $p_A^E - p_B^E$ and the change in each newspaper's circulation implied by it, given that the equilibrium advertising strategies are characterized by Equation 3.

For the advertising profits in A13 and in A17, we have

$$\left.\frac{d\Phi_A^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial\Phi_A^E}{\partial m} + \frac{\partial\Phi_A^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{\theta(rv(3+a)-6k)^2}{12r(3+a)} > 0,$$

$$\left.\frac{d\Phi_B^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial\Phi_B^E}{\partial m} + \frac{\partial\Phi_B^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{\theta(rv(3-a)-6k)^2}{12r(3-a)} > 0.$$

Due to B's shrinking circulation, we also have

$$\left.\left(\frac{d\Phi_A^E}{dm} - \frac{d\Phi_B^E}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{\theta a\left(r^2v^2(9-a^2)-36k^2\right)}{6r(3-a)(3+a)} > 0.$$

Finally, for the profits from advertising in per-reader terms, $\phi_n^E = \frac{\Phi_n^E}{R_n^E}, n \in \{A,B\}$, $k < \frac{rv(3-a)}{6}$ again allows to us conclude that we must have

$$\left.\frac{d\phi_A^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial\phi_A^E}{\partial m} + \frac{\partial\phi_A^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{\theta(rv(3+a)-6k)^2}{2r^2(3+a)^2} > 0,$$

$$\left.\frac{d\phi_B^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial\phi_B^E}{\partial m} + \frac{\partial\phi_B^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{\theta(rv(3-a)-6k)^2}{2r^2(3-a)^2} > 0,$$

and

$$\left(\frac{d\phi_A^E}{dm} - \frac{d\phi_B^E}{dm}\right)|_{m=0,x=\frac{2at}{3}} = \frac{12\theta ak\left(rv\left(9-a^2\right)-18k\right)}{r^2(3+a)^2(3-a)^2} > 0.$$

Part 4. The derivatives of the newspapers' profits π_A^E and π_B^E are

$$\left.\frac{d\pi_A^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial\pi_A^E}{\partial m} + \frac{\partial\pi_A^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = -\theta k\left(v - \frac{6k\left(a^2-2a+9\right)}{r(3-a)^2(3+a)}\right) < 0,$$

$$\left.\frac{d\pi_B^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial\pi_B^E}{\partial m} + \frac{\partial\pi_B^E}{\partial x^E}\frac{dX_m}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = -\theta k\left(v - \frac{6k\left(a^2+2a+9\right)}{r(3-a)(3+a)^2}\right) < 0.$$

Thus, both newspapers' profits decrease as m increases. Furthermore, we have

$$\left.\left(\frac{d\pi_A^E}{dm} - \frac{d\pi_B^E}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{12\theta ak^2\left(3+a^2\right)}{r(3-a)^2(3+a)^2} > 0.$$

Thus, π_A^E decreases by a smaller amount than π_B^E, as m increases. As $\pi_A^E > \pi_B^E$ holds if $m = 0$, the ranking of the profit changes remains unaltered if the changes are measured in terms of the initial profit levels, rather than in absolute terms.

Proof of Theorem 3

It is a relatively straightforward exercise to verify that interior equilibria of the type that we study exist for a suitably small neighborhood of m_0 and r_0, with $m > 0$. As in the proof of Parts 2, 3, and 4 of Theorem 2, we generally proceed by identifying the signs of the relevant derivatives, evaluated at $m = 0, x = \frac{2at}{3}$, and by invoking continuity of the derivatives, which implies local persistence of the sign.

Part 1. Let us set, for notational convenience,

$$P_n^E = \frac{dp_n^E}{dr} = \frac{\partial p_n^E}{\partial r} + \frac{\partial p_n^E}{\partial x^E}\frac{\partial X_m}{\partial r}, n \in \{A, B\}.$$

In the absence of advertising, changes in the aggregate circulation have no strategic effects, as $P_n^E|_{m=0,x=\frac{2at}{3}} = 0, n \in \{A, B\}$ holds. Furthermore, we have

$$\left.\frac{dP_A^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial P_A^E}{\partial m} + \frac{\partial P_A^E}{\partial x^E}\frac{\partial X_m}{\partial m}\right)\right|_{m=0,x=\frac{2at}{3}} = -\frac{36\theta k^2\left(a^2 - 2a + 9\right)}{r^3(3-a)^2(3+a)^2} < 0,$$

$$\left.\frac{dP_B^E}{dm}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial P_B^E}{\partial m} + \frac{\partial P_B^E}{\partial x^E}\frac{\partial X_m}{\partial m}\right)\right|_{m=0,x=\frac{2at}{3}} = -\frac{36\theta k^2\left(a^2 + 2a + 9\right)}{r^3(3-a)^2(3+a)^2} < 0.$$

Thus, for small values of m, both p_A^E and p_B^E decrease as r increases. The ranking of $\frac{dP_A^E}{dm}$ and in $\frac{dP_B^E}{dm}$ is already evident from the previous equations, noting that

$$\left.\left(\frac{dP_A^E}{dm} - \frac{dP_A^E}{dm}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{144\theta a k^2}{r^3(3-a)^2(3+a)^2} > 0, \tag{A24}$$

is nevertheless useful for future reference—see the proof of Theorem 4.

Part 2. For the threshold levels of the willingness to pay, we have

$$\left.\frac{dq_A^E}{dr}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial q_A^E}{\partial r} + \frac{\partial q_A^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = -\frac{6k}{r^2(3+a)} < 0,$$

$$\left.\frac{dq_B^E}{dr}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial q_B^E}{\partial r} + \frac{\partial q_B^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = -\frac{6k}{r^2(3-a)} < 0.$$

Inspection of the equations is sufficient to conclude that the decrease in q_B^E is greater than the decrease in q_A^E. For the total profits from advertising, whose expressions are given by A13 and by A17, we have

$$\left.\frac{d\Phi_A^E}{dr}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial \Phi_A^E}{\partial r} + \frac{\partial \Phi_A^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = \theta\left(\frac{v^2(3+a)}{12} - \frac{3k^2}{r^2(3+a)}\right) > 0,$$

$$\left.\frac{d\Phi_B^E}{dr}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial \Phi_B^E}{\partial r} + \frac{\partial \Phi_B^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = \theta\left(\frac{v^2(3-a)}{12} - \frac{3k^2}{r^2(3-a)}\right) > 0.$$

The changes can immediately be ranked by comparing the previous expressions.

For the profits from advertising in per-reader terms, $\phi_n^E = \frac{\Phi_n^E}{R_n^E}, n \in \{A, B\}$, we have

$$\left.\frac{d\phi_A^E}{dr}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial \phi_A^E}{\partial r} + \frac{\partial \phi_A^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{6\theta k}{r^2(3+a)}\left(v - \frac{6k}{r(3+a)}\right) > 0,$$

$$\left.\frac{d\phi_B^E}{dr}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial \phi_B^E}{\partial r} + \frac{\partial \phi_B^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{6\theta k}{r^2(3-a)}\left(v - \frac{6k}{r(3-a)}\right) > 0.$$

The ranking of the changes is based on

$$\left.\left(\frac{d\phi_A^E}{dr} - \frac{d\phi_B^E}{dr}\right)\right|_{m=0,x=\frac{2at}{3}} = -\frac{12\theta ak}{r^2(3-a)(3+a)}\left(v - \frac{36k}{r(3-a)(3+a)}\right) < 0.$$

Part 3. The derivatives of the newspapers' profits are

$$\left.\frac{\partial \pi_A^E}{\partial r}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial \pi_A^E}{\partial r} + \frac{\partial \pi_A^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{t(3+a)^2}{18} > 0,$$

$$\left.\frac{\partial \pi_B^E}{\partial r}\right|_{m=0,x=\frac{2at}{3}} = \left.\left(\frac{\partial \pi_B^E}{\partial r} + \frac{\partial \pi_B^E}{\partial x^E}\frac{\partial X_m}{\partial r}\right)\right|_{m=0,x=\frac{2at}{3}} = \frac{t(3-a)^2}{18} > 0.$$

The profit changes can immediately be ranked by comparing the previous expressions.

Proof of Theorem 4

Part 1. A necessary condition for surplus maximization is clearly existence of a reader with address $z \in [0, 1]$ such that any reader whose address is smaller than (greater than) z is a reader of A (of B)—except possibly for a set of readers with measure 0. It is convenient to write A's readership R_A as the product of z and the aggregate circulation r. We can then write the surplus directly generated by the newspapers' content, net of the printing and distribution costs, as

$$\begin{aligned}\rho &= r\left(\int_0^a (u - t(a-s))ds + \int_a^z (u - t(s-a))ds + \int_z^1 (u - t(1-s))ds - c\right)\\ &= r\left(u - c - t\left(\frac{1}{2} + a^2\right) + t(1+a)z - tz^2\right).\end{aligned}$$

The newspapers' advertising strategies (Equation 3) maximize the total surplus produced by the ads, given the newspapers' circulations. Thus, the ("interior") optimized surplus from advertising, for any given z, can be written as

$$\alpha = m\left(\int_{\frac{k}{rz}}^{v}(rzw-k)dw + \int_{\frac{k}{r(1-z)}}^{v}(r(1-z)w-k)dw\right) = \frac{mv(rv-4k)}{2} + \frac{k^2m}{2rz(1-z)}.$$

Using $\tau = \alpha + \rho$ to denote the net total welfare, we can then write the first- and second-order conditions for an interior optimum as

$$\frac{d\tau}{dz} = rt(1+a-2z) + \frac{k^2m(2z-1)}{2rz^2(1-z)^2} = 0 \tag{A25}$$

and

$$\frac{d^2\tau}{dz^2} = \frac{k^2m\left(3z^2-3z+1\right)}{rz^3\left(1-z\right)^3} - 2rt < 0, \tag{A26}$$

If $m = 0$, both A25 and A26 are verified at $z = \frac{1+a}{2}$. Furthermore, using A25, we obtain

$$\left.\frac{dz}{dm}\right|_{m=0,z=\frac{1+a}{2}} = -\left.\frac{\dfrac{\partial\left(rt(1+a-2z)+\dfrac{k^2m(2z-1)}{2rz^2(1-z)^2}\right)}{\partial m}}{\dfrac{\partial\left(rt(1+a-2z)+\dfrac{k^2m(2z-1)}{2rz^2(1-z)^2}\right)}{\partial z}}\right|_{m=0,z=\frac{1+a}{2}}$$

$$= \frac{4ak^2}{r^2t(1-a)^2(1+a)^2}.$$

The Implicit Function Theorem guarantees then existence of a continuously differentiable, increasing function z^*, defined over a conveniently small right neighborhood of $m = 0$, identifying the values of z that solve A25. Continuity of z^* implies that A26, which holds at $m = 0$, will also hold if suitably small values of m are considered, together with the associated values of z^*. An argument similar to the one used in the proof of Part 1 of Theorem 2, to establish uniqueness of the interior equilibrium associated with X_m, allows then to establish that z^* is the unique relevant solution of A25, at least locally. Furthermore, if both newspapers have a positive circulation, Equation 1 yields

$$\frac{d\left(\frac{R_A^E}{r}\right)}{dx} = -\frac{1}{2t}. \tag{A27}$$

Using A23, we can then write

$$\frac{d\left(\frac{R_A^E}{r}\right)}{dm} = -\frac{1}{2t}\frac{dX_m}{dm} = \frac{36\theta ak^2}{t(3-a)^2(3+a)^2},$$

and, therefore,

$$\frac{dz^*}{dm} - \frac{dz^E}{dm} = \frac{4ak^2\left((3-a)^2(3+a)^2 - 9\theta(1-a)^2(1+a)^2\right)}{r^2t(3-a)^2(1-a)^2(1+a)^2(3+a)^2}.$$

The previous expression is decreasing in θ for every admissible value of a; furthermore, we have

$$\left((3-a)^2(3+a)^2 - 9\theta(1-a)^2(1+a)^2\right)\Big|_{\theta=1} = 8\left(3-a^2\right)\left(3+a^2\right) > 0.$$

Hence, $\frac{dz^*}{dm} - \frac{dz^E}{dm} > 0$ certainly holds, and the distance between the two circulations increases as m increases, at least for relatively small values of m.

Part 2. Setting, for notational convenience,

$$Z = \frac{dz^*}{dr} = -\frac{\dfrac{\partial\left(rt(1+a-2z) + \dfrac{k^2m(2z-1)}{2rz^2(1-z)^2}\right)}{\partial r}}{\dfrac{\partial\left(rt(1+a-2z) + \dfrac{k^2m(2z-1)}{2rz^2(1-z)^2}\right)}{\partial z}},$$

we have $Z|_{m=0,z=\frac{1+a}{2}} = 0$ and

$$\left.\frac{dZ}{dm}\right|_{m=0,z=\frac{1+a}{2}} = \left.\left(\frac{\partial Z}{\partial m} + \frac{\partial Z}{\partial z}\frac{\partial z^*}{\partial m}\right)\right|_{m=0,z=\frac{1+a}{2}} = -\frac{4ak^2}{r^3t(1-a)^2(1+a)^2} < 0.$$

Hence, the planner would decrease the relative circulation of A, as r increases, at least for conveniently small values of m and r. As we know from Theorem 3, the equilibrium relative circulation of A also decreases, if r increases. Using A24 and A27, we have

$$\left.\left(\frac{dZ}{dm} - \frac{1}{2t}\left(-\left(\frac{\partial P_A^E}{\partial m} - \frac{\partial P_A^E}{\partial m}\right)\right)\right)\right|_{m=0,x=\frac{2at}{3}}$$
$$= \frac{4ak^2\left(18\theta(1-a)^2(1+a)^2 - (3-a)^2(3+a)^2\right)}{r^3t(3-a)^2(1-a)^2(1+a)^2(3+a)^2}.$$

If $\theta = 1$, $\left(18\theta(1-a)^2(1+a)^2 - (3-a)^2(3+a)^2\right)$ evaluates to -63. The previous expression is therefore negative: The relative circulation of A decreases by more than the planner would wish it to decrease, and the difference between the equilibrium and the optimal partition of the readers grows larger, as r increases.